Sunday Academy
(2011-2012)
Boolean Algebra

Lesson Background

Logic circuits are the basis for modern digital computer systems. To appreciate how computer systems operate one will need to understand digital logic and Boolean algebra. In this lesson, students will be introduced to various properties of Boolean algebra and will learn how digital circuits operate. In Electrical and Computer Engineering careers, minimizing terms and expressions can be important because electrical circuits often consist of individual components that implement each term or literal for a given expression. Minimizing the expression allows the designer to use fewer electrical components and, therefore, can reduce the cost of the system.

Objectives

Students will

- understand and perform various binary operations such as addition (OR), multiplication (AND), and negation (NOT)
- apply Boolean properties
- develop logical truth table based on Boolean equations
- build and test the Boolean digital lock circuit using a trainer kit, and verify its truth table.

North Dakota Standards

11-12.2.2. Abilities to do scientific inquiry, select and use appropriate instruments, measuring tools, and units of measure to improve scientific investigations.
11-12.2.7. Design and conduct an independent investigation.
9-10.5.14. Mathematical modeling, draw conclusions about a situation being modeled.

Timeline

11:00 – 11:30: Cultural Connection/brief Introduction
11:30– 12:00 : Power point Introduction to Boolean Algebra, Bread Board overview.
12:00 – 12:45: Lunch
12:45 – 1:30: Activity 1, Basic Gates using Boo1e0: AND, OR, NOT
1:30 –2.00: Activity 2, Applying Properties of Boolean Algebra
2:00–2:45: Activity 3: Build the function described in Activity 2
2:45 -3:00: Summary of Boolean algebra

Materials Required: Digital Logic Trainer Kit, B001e0 cards, Dominoes sets

Background Information on Logical Gates

The idea and implementation of logic gates has developed greatly over the past 100 years. Gates can be used in almost any situation to display the possible outcomes when multiple inputs are present. The first logic gates were developed around 1837 by George Boole and were of mechanical form of switches. For most of that century, these gates went through much development until Nikola Tesla developed circuit-based logic in 1898. Furthermore, Boolean algebra was introduced to circuit analysis during the mid-19th century and shaped the way people interpret this logic today. There are many different types of logic in electronics, where gates are made up of many different devices such as diodes, relays, transistors, and most importantly, integrated circuits (ICs). These ICs, which can be mounted on a circuit board or used in other applications such as breadboards, have logic gates built-in and can be used easily with the help of datasheets. Common ICs have approximately eight or 14 pins and provide the user with three to four of the same logic gate.

Here are some examples of logical relationships:

1. "If you wash the dishes and clean your room, you can go to the party." Your kid has washed the dishes but not cleaned the room. Does your kid get to go to the party under this rule? (AND)

2. "I will let you go to the party if you wash the dishes or clean your room." Your kid has cleaned their room but the dishes are still dirty. Does your kid get to go to the party under this rule? (OR)

3. Same rule as #2, but your kid both washed the dishes and cleaned up their room - do they get to go? (Inclusive or, OR, includes cases where both conditions are true)

Things to remember and note down

Logic Gates are drawing that represent these logical relationships (AND, OR, NOT) with shapes. These three gates are called Basic Gates. Logic gates are drawn with

- a shape representing the function of the gate (AND, OR, NOT)
- two input lines on the left-hand side (sometimes more, but not here), and letters to identify the inputs
- one output line on the right-hand side, and a letter to identify the output

The logical function is made explicit with a truth table. In the truth table, 0 represents false and 1 represents true. Here are the most important examples.

Input and output relationships:

If there are n inputs, then there are 2^n possible output combinations**.**

Always start at the left with the inputs, and end at the right with the outputs.

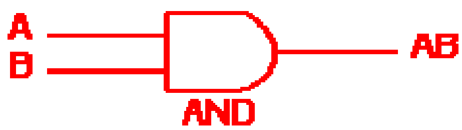
Digital systems are said to be constructed by using logic gates. The basic gates are the AND, OR, NOT gates. The basic operations of gates are described with the aid of tables called truth tables.

Activity 1:

Part 1: Introduction to Basic Gates (AND, OR, NOT)

Students will be filling out the outputs of these gates

AND gate

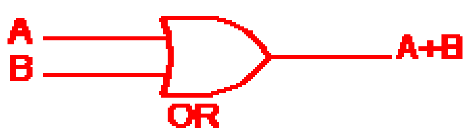


2 Input AND gate		
A	B	A.B
0	0	
0	1	
1	0	
1	1	

Logical AND Circuit *Truth Table of AND*

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high. A dot (.) is used to show the AND operation (i.e. A.B). Bear in mind that this dot is sometimes omitted (i.e. AB). The output of an AND gate is true if (1) input A is true AND (2) input B is true, otherwise the output is false.

OR gate

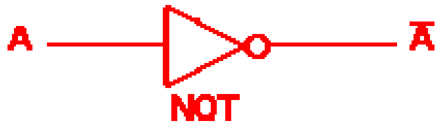


2 Input OR gate		
A	B	A+B
0	0	
0	1	
1	0	
1	1	

Logical OR Circuit Truth Table of OR

The OR gate is an electronic circuit that gives a high output (1) if **one or more** of its inputs are high. A plus (+) is used to show the OR operation. The output of an OR gate is true if (1) input A is true OR (2) input B is true; otherwise the output is false (*inclusive* or, result is true if both A and B are true).

NOT gate



Logical NOT Circuit

NOT gate	
A	$\bar{A}$
0	
1	

Truth Table of OR

The NOT gate is an electronic circuit that produces an inverted version of the input at its output. It is also known as an *inverter*. If the input variable is A, the inverted output is known as NOT A. This is also shown as A', or A with a bar over the top, as shown at the outputs.

Part 2 of Activity 1:

Complete the Truth Table of the Boolean equation AB+C

A	B	C	AB	Output
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

List the Conditions of A, B and C such that the output is 1. For examples, you may say the output is 1 when A=1, B=0, C=1.

Activity 2: Understanding Properties of Boolean Algebra

The following are some 25 properties of Boolean Algebra. Students will be verifying these properties from the list using its truth table.

P1. A+B = B+ A	P2. AB=BA	Commutative
P3. A+(B+C)=(A+B)+C	P4. A(BC)=(AB)C	Associative
P5. A+0=A	P6. A.1=A	Identity
P7. A+1=1	P8. 1.A=A	
P9. 1+A=1	P10. A.0=0	
P11.A+A’=1	P12. A.A’=0	Complement
P13. A’+A=1	P14. A’.A=0	
P15. A+A=A	P16. A.A=A	
P17. (A’)’=A	P18. A(B+C)=AB+AC	
P19.A+BC=(A+B)(A+C)	P20. AB +AB’=A	
P21. A+A’B=A+B	P22. A+AB=A	
P23. A(A+B)=A	P24.A(A’+B)=AB	
P25.(AB)’=A’+B’		

Table. 1 Boolean Properties

Part 1: Verify the following properties and fill out the output with either 1 or 0.

Verify P18. A(B+C)=AB+AC

Let us do Left Hand side (L.H.S) first and then do the right hand side (R.H.S)

A	B	C	B+C	**L.H.S A(B+C)**	AB	AC	**R.H.S (AB+AC)**
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Are the LH.S and R.H.S of P18 are equal?________________________________

Verify P.25 (AB)’=A’+B’

A	B	AB	**L.H.S (AB)’**	A’	B’	**R.H.S A’+B’**
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Are the LH.S and R.H.S of P25 are equal?________________________________

Verify Property 17 (A’)’=A

A L.H.S	A’	(A’)’ R.H.S
0		
1		

Are the LH.S and R.H.S of P17 are equal?________________________________

Pick any two other properties from the table and show the output:

Part 2: Use of Boolean properties in digital circuit design

A. Draw the Logical circuit for the equation AA+AC+AB+BC. How many gates did you use?

B. Apply the Boolean properties to simplify the expression. Are you getting A+BC terms?

C. Draw the logical circuit for the simplified expression obtained in Part 2 (B). How many gates did you use to build the circuit?

D. Complete the truth table for A A+A C+A B+B C

A	B	C	A A	A C	A B	B C	Output AA+AC+AB+BC
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

E. Complete the truth table for the simplified expression

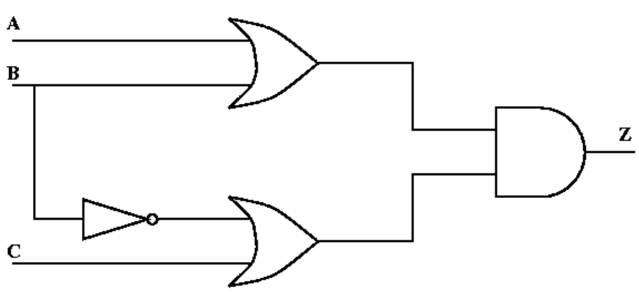
A	B	C	BC	Outputs A+BC
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

F. Do the truth tables in D and E yield same result? Compare the number of gates you used in A and C. Did you see any difference? What do you think Boolean algebra does now?

Part 3: Problem Solving

Test your understanding on writing Boolean equations.

1. What is the output equation at z?.



Z=____

2. Complete the truth table

A	B	C	A+B	C’	(A+B)C’
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

3. Draw the logical Circuit for (A+B)C’

4. Back track from the truth table to develop the equation. List the properties you used that helped in recovering the equation back.

Activity 3: Build Logic Circuit for a DIGITAL KEY LOCK. Assume the digital lock as 3 keys A, B and C. The lock should open only for the following key sequence.

ACTIVITY 3

Build a digital lock circuit such that the lock opens (meaning LED glows- ON state) only for the following sequence. For all other sequences of A, B and C keys, the digital lock remain closed (LED OFF STATE). Students will be using the Digital Lab Training Kit for the activity. In order to build the circuit, students need to follow the procedures outlined below for the circuit to work:

Digital Lock

A	**B**	**C**	**Digital Lock**
0	1	0	Opens, LED glows
1	0	0	Opens, LED glows
1	1	0	Opens, LED glows

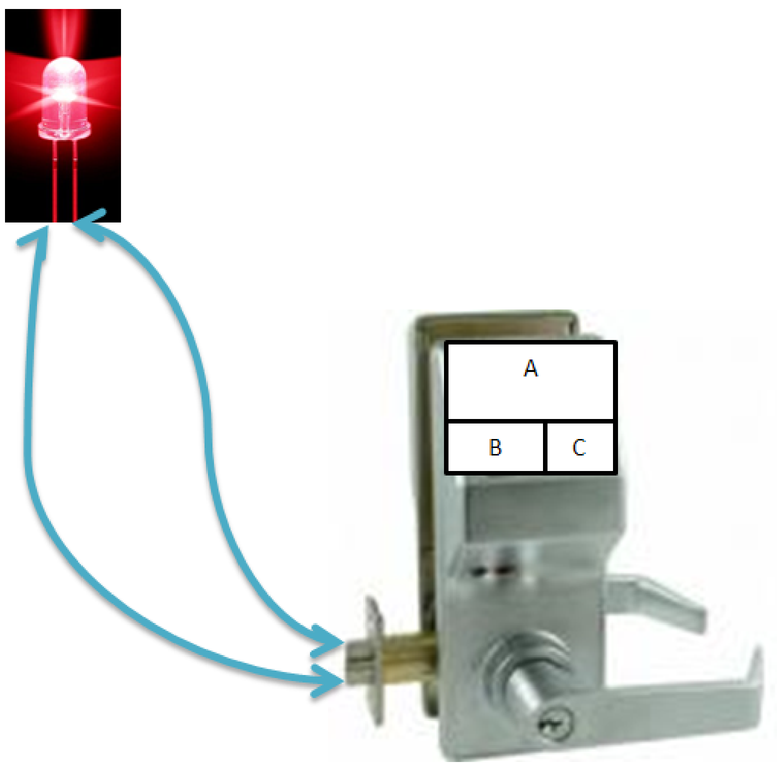


Figure 3 Digital Lock

Procedure:

Step 1: Turn on the +5V power switch located at the left side of the kit and also makesure the actual power button of the kit is ON.

Step 2: Insert all AND, OR, and NOT IC chips on the board as shown.

Step 3: Take a wire. Place one end of the wire to the 7^{th} pin of OR gate and the other end to the 7^{th} pin of NOT gate.

Step 4: Take a wire. Place one end of the wire to the 7^{th} pin of NOT gate to 7^{th} pin of AND gates.

Step 5: Take a wire. Place one end of that wire to the 7^{th} pin of AND gate and the other end to Ground connection located at the right side of the kit.

Step 6: Take a wire. Place one end of the wire to the 14^{th} pin of OR gate and the other end to the 14^{th} pin of NOT gate.

Step 7: Take a wire. Place one end of the wire to the 14^{th} pin of NOT gate to 14^{th} pin of AND gate.

Step 8: Take a wire. Place one end of that wire to the 14^{th} pin of AND gate and the other end to + 5v power connection located at the left side of the kit.

Step 9: Keep 3 wires ready. One of the wires should be connected to Pin 1 and its other end to switch 1. The second wire should connect Pin 2 of 7432 to Switch 2 and third wire should connect Pin 1 of 7404 to switch 3. The switches are located in the bottom side of the kit.

Step 10: Take a wire. Connect the pin 3 of 7432 to pin 1 of 7408.

Step 11: Take a wire. Connect the pin 2 of 7404 to pin 2 of 7408.

Step 12: You are almost done. You should now connect Pin 3 of 7408 to any of the LED pins. You can place your wire at either of the two slots. You may connect the other end to a BUZZER, if you wish to hear any sound for 1.

Step 13: Try all combinations of inputs A, B and C to verify the desired output.

Step 14: You are done.

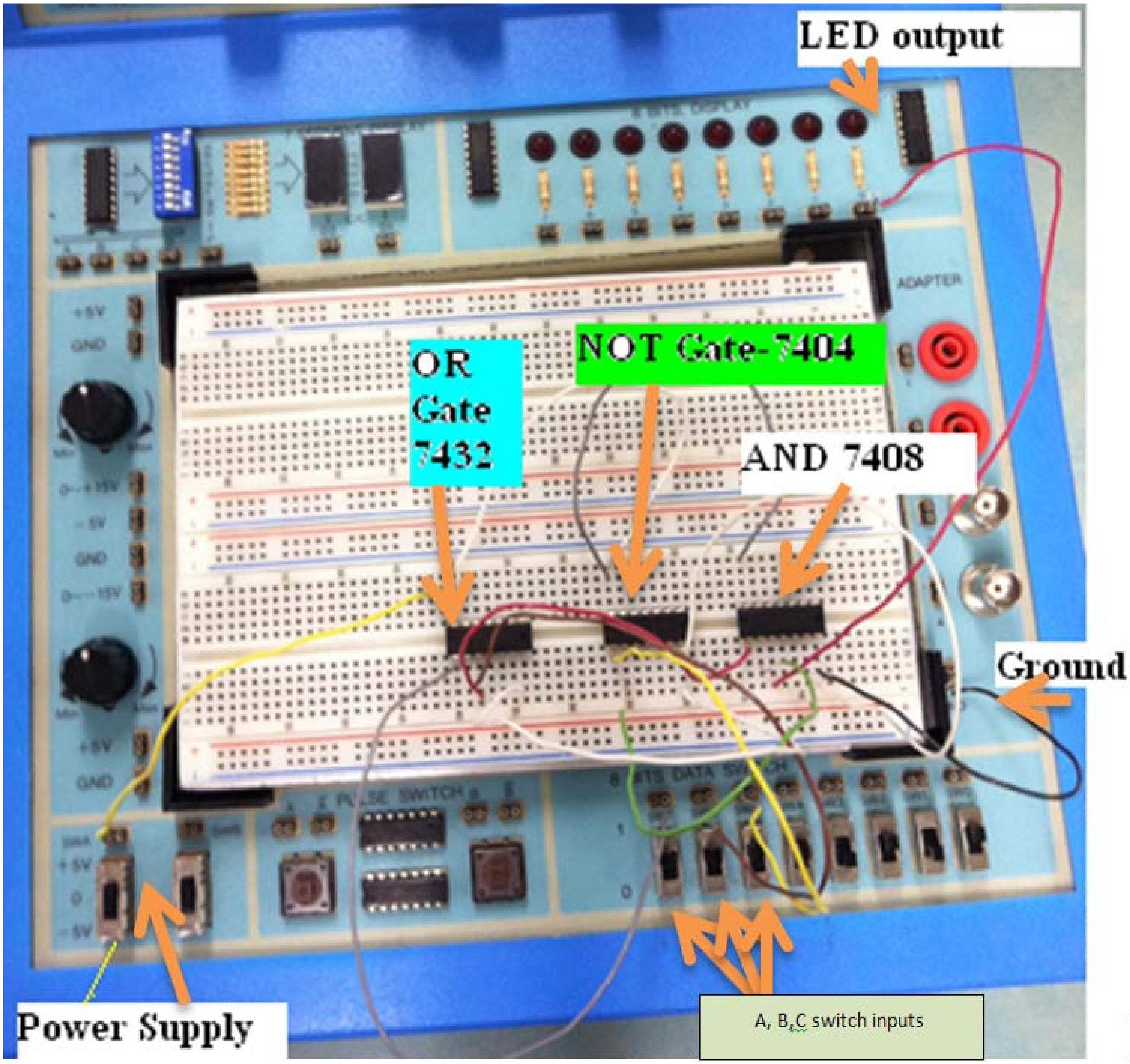


Figure.1 Circuit Layout for Boolean function (A+B)C’

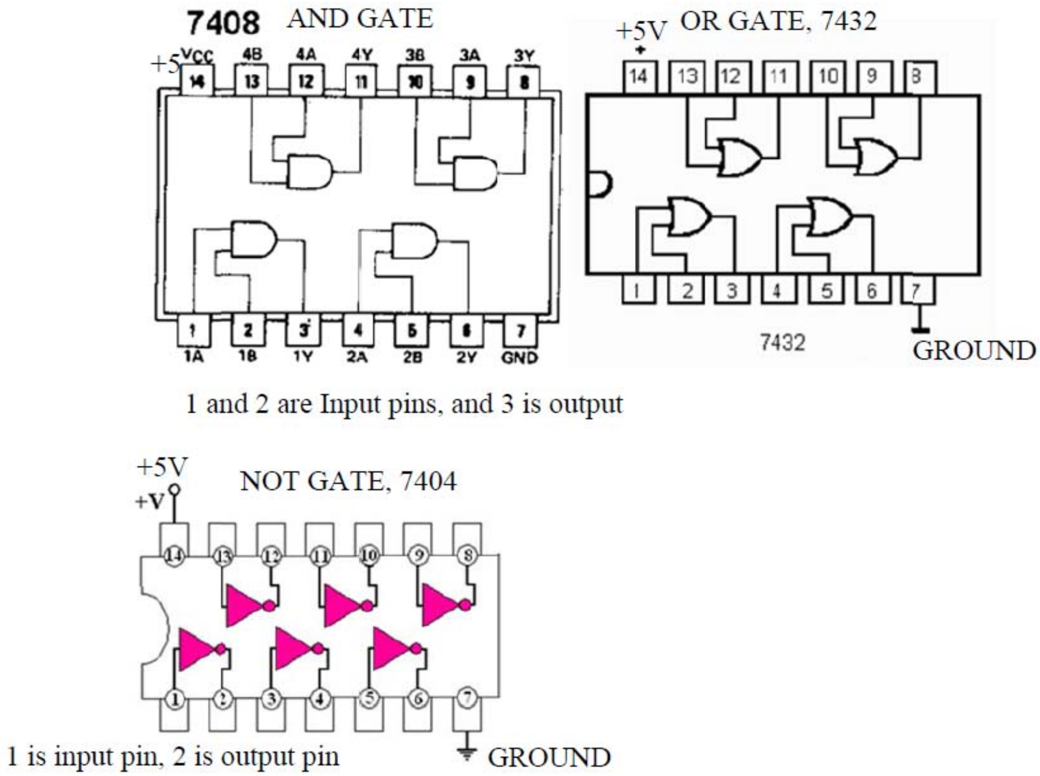


Figure.2 Pin Layout for AND (7408), OR (7432), NOT (7404) Gates

Summary of the lesson plan

1. Boolean Variables can take either __________ or ____________
2. The basic types of logical gates are____
3. If A=1 and B=0, then the truth value of the equation B’A=_______________
4. The property 1+ A equals to________________
5. The binary addition of 1+1 yields _______________________________________
6. The binary multiplication of 1.0 yields____________________________________
7. An equivalent representation for the Boolean expression A' + 1 is__________ (A or A’ or 1 or 0)
8. For four inputs logical AND Gate (A, B, C and D) how many possible combinations of output are there? List the combinations and mention its output.