



## NATURE TCC Sunday Academy Program

2017-2018

# Game Theory

## How to win the game!

### INSTRUCTOR VERSION

#### Description:

Game theory is the mathematics of strategy. John Von Neumann, a Hungarian born mathematician, is considered the father of game theory. The sometimes poker player was inspired by the game to develop this branch of mathematics in the late 1920's.

Game theory studies decisions made in an environment where various participants interact. It takes into account the variety of choices that each person has at each point in the process and determines optimal behavior when weighed against the costs and the benefits of each action.

Unless a one-player game, a player's choices will be influenced by the actions and behaviors of other participants. The games studied by game theorists consist of a set of players, a set of moves, or strategies, and a specification of the payoffs (benefits) for each combination of strategies.

Games can be placed into 2 categories, games of chance and total information games. Games of chance are games whose outcomes are strongly influenced by some randomizing device. Common devices used include random decisions by humans, dice, spinning tops, playing cards, roulette wheels or numbered balls drawn from a container. Total information games are games where the players know, in advance, all the game options and player's decisions throughout the game are based on the previous player's decision. In this lesson, we will analyze the possible outcomes for the "First to Fifteen" game to determine which moves will result in a greater chance of winning the game, then apply this strategy to similar games.

#### Objectives:

1. Gather data by playing the game to determine best possible moves.
2. Use game theory to determine the best possible move for the game.
3. Apply game theory to other games of similar type.
4. Develop a winning strategy for a game.

**Materials Needed:**

- Game pieces (markers numbered with digits 1-9)– one set for each pair of students

**State Mathematics Standards:**

- 12.3.3 – Use counting strategies
- 12.3.4 – Calculate theoretical and experimental probabilities
- 12.3.7 – Draw inferences and predict outcomes from data expressed in a variety of ways
- 12.5.5 – Develop and analyze a variety of algorithms
- 12.5.7 – Use patterns and functions to model problems

**Career Connections and Applications:**

At first glance, game theory sounds trivial, but applications of game theory are extensive. Game theory can be applied to economic analysis. Every factor in the market, such as seasonal preferences, buyer choice, changes in supply and material costs can be used to describe strategies to maximize the outcome and profit. Military strategists have turned to game theory to play “war games”. Immunization procedures and vaccine or other medication tests are analyzed using game theory. Locally, casinos use game theory on a variety of levels to determine profit or loss.

**Timeline:**

- 11:00 – 11:30 Cultural Connection
- 11:30 – 11:45 Lesson Introduction
- 11:45 – 12:30 Activity 1 – Gather Data for “First to Fifteen” game
- 12:30 – 1:00 Lunch
- 1:00 – 1:30 Activity 2 – Analyze Game Data to determine winning strategy
- 1:30 – 2:00 Activity 3 – Theoretical Probability for “First to Fifteen”
- 2:00 – 2:30 Activity 4 – Magic Square
- 2:30 – 2:45 Activity 5 – Tic Tac Toe
- 2:45 – 3:00 Evaluation and Conclusions

**Cultural Connection:****The Hand Game**

This game is played among eighty-one Indian tribes of the United States. The game bears different names in the various languages of these tribes. Hand Game is a descriptive term and not a translation of any native name; it refers to the fact that the object is held in the hand during the play. The following form of this game is the way it was formerly played among the Nez Perce Indians of the State of Idaho. Lewis and Clark, who were the first white men to

record their meeting with these Indians, mention this game, and Capt. Bonneville gives an account of it when he visited the tribe during the third decade of the last century.

*Properties.*—A bone or wooden bead about two inches in length and half an inch in thickness; thirty counting sticks (these are sometimes spoken of as arrows, and there are indications that they were once arrows—the arrows of the twin gods); a mat oblong in shape; two logs or pieces of board about the length of the mat, and as many sticks (to be used as drum-sticks) as players can sit on one side of the mat.

*Directions.*—The mat should be laid east and west, the logs or boards put on the north and south edges and the counting sticks placed in two piles of fifteen each on the ends of the mat. The players sit on the ground, a row on each side of the mat to the north and south. Lots are drawn to decide which side shall have the bead "in hand." The Leader and the singers must always stand behind the row of players who have the bead "in hand." The opposite side must have the drum-sticks and beat on the log or board in time with the singers.

When the players are seated in two rows, one on each side of the mat, the Leader hands the bead to a player on the side that has drawn the right to have the bead "in hand," and then takes his place beside the singers, who stand behind that row, and starts the following song. All in that row join in the singing.

#### Native American Songs – The Hand Game Song



The players on the opposite side, who are to guess who is hiding the bead, at once begin to beat the time of the song on the log or board that is in front of them, on the edge of the mat, and at the same time they must watch the other side where the players are trying to pass the bead from one hand to the other and from one person to another without exposing the bead to view. In all these actions the movements of hands, arms and body must be rhythmical and in time with the song. All the players in the row that has the bead "in hand" must act as if each one either had the bead or was trying to pass it on, whether he actually has the bead or does not have it.

When one on the opposite side thinks he detects the whereabouts of the bead and is willing to risk a guess, he points his drum-stick to the hand he thinks has the bead and cries, "Hi-i!" and the hand indicated must be immediately opened so that all may see whether the guess is correct or not. If the bead is seen to be in the opened hand, the Leader calls out, "Success!" and goes to the pile of counting sticks belonging to the side of the guesser, takes one and stands it in the ground in front of the successful guesser. The Leader then hands the bead to the player who has won and proceeds to gather the drum-sticks and distribute them to the players on the opposite side. The singers pass around and take their places behind the row of players who now have the bead "in hand." When all are in readiness, the Leader starts the song again and the players begin their movements of secretly passing the bead, while the other side beat time with their drum-sticks on the log or board in front of them. The side that has the bead "in hand" always does the singing, led by the Leader and singers, who must stand at the rear of the row having the bead.

If a guess is incorrect the Leader goes to the pile of counting sticks that belongs to the side which has the drum-sticks, takes a counting stick and thrusts it in the ground in front of the row opposite to the guesser; that means one lost to his side. The bead in that instance remains on the same side until it is won by the opposite side through a successful guess.

In this manner the game goes on until one side or the other has won all the thirty counting sticks and become the victor in the game.

**Lesson Introduction:**

View PowerPoint presentation describing game theory, winning strategy of a game, outcomes and probability.

# Total Information Games

## Activity 1: Gather data to “First to Fifteen:” game

### The Rules:

1. Nine markers are labeled with a digit from 1 to 9. No digit is repeated.
2. Taking turns, two players remove the markers from the table.
3. The winner is the **first** player to obtain amongst their markers, three that sum to 15.
4. The three that sum to 15 DO NOT have to be the 1<sup>st</sup> 3 selected, any combination of 3 wins.
5. It is possible to have more than three markers, but exactly 3 must sum to 15.
6. It is possible that neither player will win.
7. After each win, circle the 3 winning digits from the winning players hand.

Pairs of students will play the game 20 times and record each person’s selections in the exact order chosen. Each play will keep track of their own and their opponents selections in their handout.

Your Name: \_\_\_\_\_

Opponents Name: \_\_\_\_\_

Game	Markers					Win y/n
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

Game	Markers					Win y/n
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						

## Activity 2: Analyze Game Data to determine winning strategy

Combine your results with your partner's to determine the number of times each marker was included in the **winning** hand.

Marker	1	2	3	4	5	6	7	8	9
# of times in winning hand									

Write your results on the board to combine with the entire class.

1. Based on the entire class results, can you identify a marker you would most likely want to have in your hand?

**Results will come from student data.**

2. If so, which one? And why?

**Answers will be based on student data.**

## Activity 3 – Theoretical Probability for “First to Fifteen”

Write down all the possible sets of three unique digits (from 1 to 9) that add up to 15.

1-5-9 1-6-8 2-4-9 2-5-8 2-6-7 3-4-8 3-5-7 4-5-6

Make a table showing how often each digit is used.

Digit	1	2	3	4	5	6	7	8	9
# of times used	2	3	2	3	4	3	2	3	2

Based on this information, what is the best first move? Explain your reasoning.

**Choose the number 5, it is used in the most winning combinations.**

#### Activity 4– Magic Square

Arrange the digits 1 to 9 into the magic square so that each row, column and diagonal sums to 15.

4	9	2
3	5	7
8	1	6

1. Explain how this game is the same as playing “First to Fifteen”.

The center square is used in 4 rows or diagonals.

The corner squares are used in 3 rows or diagonals.

The middle squares are used in 2 rows.

2. Explain a winning strategy for playing Magic Square.

Place the 5 in the center as it is used in 4 winning combinations.

Place the 2,4,6,8 in the corners as they are used in 3 winning combinations.

Place the 1,3,7,9 in the middle squares as they are used in 2 winning combinations.

#### Activity 5– Tic Tac Toe

On separate paper, with your partner, play the game Tic Tac Toe 20 time and record your results as Player 1 wins, Player 2 wins, or Ties.

Record your results on the board with the entire class results.

1. Explain how this game is the same as playing “Magic Square”.

Analyze the 9 squares to determine which square is used the most winning combinations or rows or diagonals.

2. Explain a winning strategy for playing Tic Tac Toe.

The player who goes first should place their mark in the center square. The player to goes 2<sup>nd</sup> should place their mark in a corner, etc.