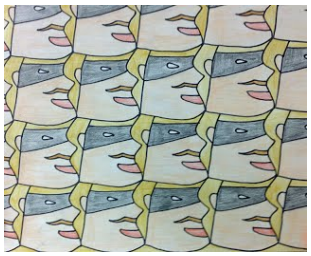
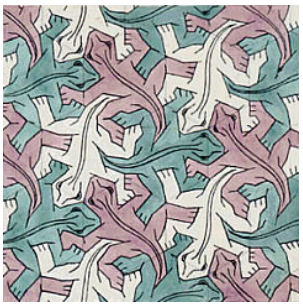


2022/23 NATURE Sunday Academy
TRANSFORMATIONAL GEOMETRY TESSELLATIONS



Description:

The word tessellation comes from the Latin Tessella, which is a small Square stone or tile used in ancient Roman mosaics. Tiles and Mosaics are common synonyms for tessellations. A plane tessellation is a pattern made up of one or more shapes, completely covering a surface without any gaps or overlaps. Basically, a tessellation is a way to tile a floor (that goes on forever) with shapes so that there is no overlapping and no gaps. Remember the last puzzle you put together? Well, that was a tessellation, only the shapes were just really weird.

A regular polygon has 3, 4, 5 or more sides and angles, all of which are equal. A regular tessellation means a tessellation made up of congruent regular polygons. Congruent means all the polygons are the same size and shape, and regular means all the sides are the same length.

According to the NCTM Standards geometry "provides an opportunity for students to experience the creative interplay between mathematics and art." This project is specifically designed to allow students to demonstrate their ability to create art using transformational geometry. Many students are familiar with the work of M.C. Escher; you will be able to create your own tessellation using computer software.

This unit integrates math with subjects such as language arts, art education and social studies. This mathematical topic can also be explored in other areas. Quilts, and the importance of quilts, can be studied through cultural activities. As a culminating activity, students will create a classroom quilt. The quilt will be a combination of a geometric shape (the square) and a geometric transformation (translation, rotation, or reflection). The classroom quilt can be considered symbolic for the class; each student could compose a story to explain his/her quilt patch.

State Standards:

HS.G-CO-1
HS.G-CO-2
HS.G-CO-3
HS.G-CO-4
HS.G-CO-5

Objectives:

1. **Communication** – students will communicate their understanding of different tessellation concepts using "math language" such as rotation, translation, reflection, square, equilateral triangle, hexagon, etc.
2. **Numeracy** – Students will engage in problem solving as they tessellate a shape using slide, rotation, and reflection techniques
3. **Personal Social Values and Skills** – students will work cooperatively in group learning activities.
4. **Independent Learning** – students will participate in exploring tessellation patterns, which will lead to independent exploration.

Schedule:

11:00 – 11:30 Cultural Connection
11:30 – 12:00 PowerPoint Introduction
12:00 – 12:30 LUNCH
12:30 – 1:00 Activity 1 Which Polygons Tessellate?
1:00 – 1:45 Activity 2 Create Tessellation using Square
1:45 – 2:30 Activity 3 Create Tessellation using Triangle
2:30 – 2:45 Activity 4 Create a Classroom Quilt
2:45 – 3:00 Evaluation and Wrap Up

MATERIALS

Poster Board Pieces 7 cm X 7 cm square (1 for each student plus a couple spares)
Poster Board Pieces 7cm equilateral triangle (1 for each student plus a couple spares)
Typing paper
Scissors for each student
Tape
Colored Pencils, Markers, etc.

VOCABULARY

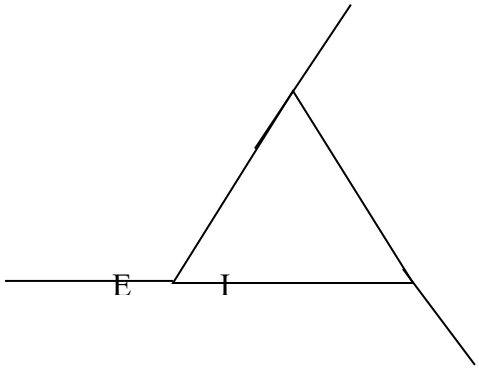
Vertex – The point where line segments meet to form a corner.
Polygon – A closed set of line segments that do not cross each other and not more than 2 segments meet at a vertex.
Translation – An action where one side is slid to the other side of a polygon so that it will match the other side exactly.
Reflection – An action where an object mirrored over a given line.
Rotation – An action where an object is turned around a given point.
Glide Reflection – A combination of a reflection and a translation on an object.
Point of Rotation – A fixed point at which some figure is rotated.

ACTIVITY #1 – Which Polygons Tessellate?

In order for a polygon to tessellate, it must fill all the gaps around a point. If you travel completely around a point, you have traveled 360°, therefore, in order for any polygon to completely fill a gap around 360°, the interior angle must be a factor of 360.

What is the measure of any interior angle in an *n* degree polygon?

To find this, we can look at the figure below.



To find the measure of interior angle *I*, we can use exterior angle *E* as a guide. If you started at *E* and walked around the triangle you would have to make 3 turns to end up back where you started. If you end up back where you started, you have traveled 360°, therefore, 360/3 = 120. Each exterior angle *E* measures 120°. Also knowing that angle *E* and angle *I* together form a straight angle 180° leads us to the fact that 180 – *E*(120) = 60. Each interior angle *I* of a triangle = 60°.

1. Does a triangle tessellate? (Does 60 divide evenly into 360?)

2. Use the procedure above to find out which other regular polygons tessellate.

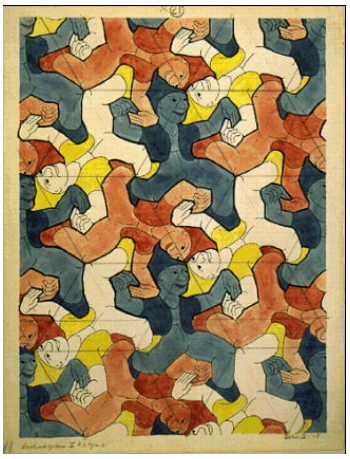
Square (4 sides) – Pentagon (5 sides) –

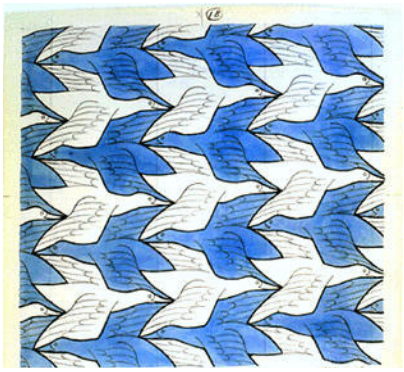
Hexagon (6 sides) – Heptagon (7 sides) –

Octagon (8 sides) – Nonagon (9 sides) –

Decagon (10 sides) – Any other regular polygons? -

WHERE DO THESE UNUSUAL TESSELLATIONS COME FROM?

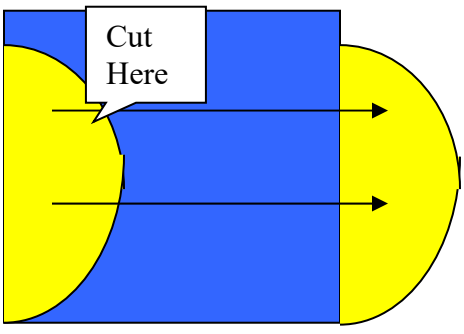




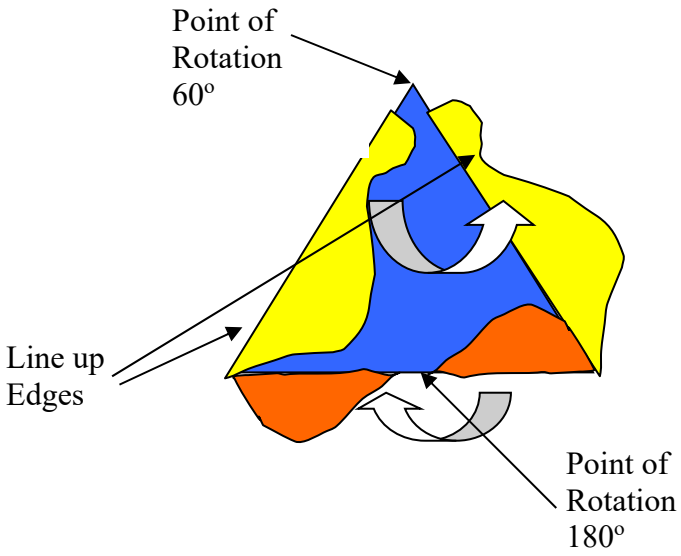
Tessellation patterns such as the ones on the previous page can be created from the regular polygons, which tessellate. There are 4 methods to creating a pattern tile to tessellate. They are translation, rotation, reflection, and glide reflection. Following is an explanation describing how to make a pattern tile using each method.

4 Types of Transformations

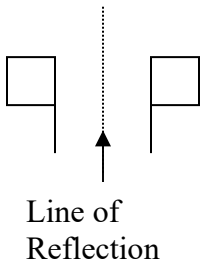
TRANSLATION - This is the easiest pattern to create and works well with a regular square. Translation means to move an object without rotating or reflecting it. Another name for translation is a slide. To create a pattern tile using a translation, you cut a piece from one side of the square and slide it straight across to the other side of the square and attach to the edge



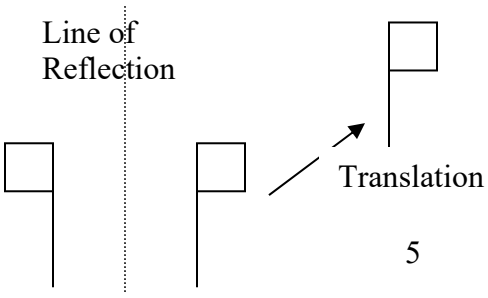
ROTATION - To rotate and object mean to spin it around a certain point. You must know the point to rotate around and what angle measure to rotate it. To create a pattern tile using an equilateral triangle, cut one section from the left side and rotate to bottom 60°. You can also rotate from one side to the same side by using the midpoint, cut a piece from one side and rotate it 180° using the midpoint as point of rotation to match up with the side it was cut from.



Reflection - A reflection is a mirror image on an object. Every reflection has a mirror line. Using a reflection produces the same results as a transformation or a rotation so we will not cover how to produce a pattern tile using a reflection.



Glide Reflection – A glide reflection is just a product of a reflection and a translation.



ACTIVITY #2 - CREATE A TESSELLATION PATTERN TILE USING A TRANSLATION AND YOUR SQUARE.

1. Using the left edge corners as starting and ending points, draw a random figure inside the square.
2. Cut out your random figure and translate (slide) straight across the square so the edges line up and carefully tape. Make sure the edges match exactly.
3. Using the bottom edge corners as starting and ending point, draw a random figure inside the square.
4. Cut out your random figure and translate (slide) straight up to the top of the square so the edges line up and carefully tape. Make sure the edges match exactly.
5. You have now created a pattern tile which will tessellate.

To make your pattern cover a surface:

1. Trace your pattern tile onto a piece of poster board, which will serve as your tracing pattern.
2. Cut out your pattern from the poster board.
3. Place your pattern piece somewhere on a sheet of typing paper and trace.
4. Translate (slide) your pattern in all directions and trace to completely fill the page. Trace your pattern completely off the edge of the paper to fill all gaps.
5. Color your tessellation.

ACTIVITY #3 - CREATE A TESSELLATION PATTERN TILE USING A ROTATION AND YOUR EQUILATERAL TRIANGLE.

1. Using the left edge corners as starting and ending points, draw a random figure inside the triangle.
2. Cut out your random figure and rotate (using the top corner as point of rotation) so your random figure matches up with the right edge and carefully tape. Make sure the edges match exactly.
3. Using the bottom left corner and the midpoint of the bottom edge as starting and ending points, draw a random figure inside the triangle.
4. Cut out your random figure and rotate (using the midpoint as point of rotation) so your random figure matches up with bottom of the triangle and carefully tape. Make sure the edges match exactly.
5. You have now created a pattern tile which will tessellate.

To make your pattern cover a surface:

1. Trace your pattern tile onto a piece of poster board, which will serve as your tracing pattern.
2. Cut out your pattern from the poster board.
3. Place your pattern piece somewhere on a sheet of typing paper and trace.
4. Rotate your pattern in all directions so edges of the pattern fit together like a puzzle and trace to completely fill the page. Trace your pattern completely off the edge of the paper to fill all gaps.
5. Color your tessellation.

ACTIVITY #4 - CONSTRUCTING A CLASSROOM QUILT.

1. Each student should have 2 tessellation pages, one made with the square, one made with the equilateral triangle, and one made with the computer program.
2. Carefully tape all tessellation pages together to form one large classroom quilt.
3. Display on classroom wall.

QUESTIONS

1. Why are an equilateral triangle, a square and a hexagon the only regular polygons that will tessellate?

2. Is it possible to construct a tessellation using a square with a rotation? Explain.

3. Is it possible to construct a tessellation using an equilateral triangle with a translation? Explain.

4. Which transformations (translation and rotation) are possible using a hexagon? Explain.

5. Create a story explaining your tessellations.